試卷一

解	分	備註
1. $\frac{a+b}{2}=\frac{4b-1}{3}$		
$3(a+b)=2(4b-1)$	$\frac{3}{2}(a+b)=4b-1$	1M	
$3a+3b=8b-2$	$\frac{3}{2}a+\frac{3}{2}b=4b-1$		
$3a+2=5b$	$\frac{3}{2}a+1=\frac{5}{2}b$	1M	給將 b 放在一邊或等價
$b=\frac{3a+2}{5}$	$b=\frac{3a+2}{5}$	1A	
	---------(3)	
2. $\frac{x^2y^{-3}}{(x^3y^{-1})^6}$		
$=\frac{x^2y^{-3}}{x^{18}y^{-6}}$	1M	給 $(ab)^m=a^mb^m$ 或 $(a^m)^n=a^{mn}$
$=\frac{y^{-3+6}}{x^{18-2}}$	1M	給 $c^{-p}=\frac{1}{c^p}$ 或 $\frac{c^p}{c^q}=c^{p-q}$
$=\frac{y^3}{x^{16}}$	1A	
	---------(3)	
3. (a) 38.2	1A	
(b) 百分誤差		
$=\frac{38.26-38.2}{38.26}\times 100\%$	1M	
$\approx 0.157\%$	1A	
	---------(3)	
4. (a) $8m^3-4m^2n$		
$=4m^2(2m-n)$	1A	
(b) $8m^3-4m^2n-18mn^2+9n^3$		
$=4m^2(2m-n)-18mn^2+9n^3$	1M	給利用 (a) 的結果
$=4m^2(2m-n)-9n^2(2m-n)$		
$=(2m-n)(4m^2-9n^2)$	1M	
$=(2m-n)(2m+3n)(2m-3n)$	1A	或等價
	---------(4)	

解	分	備註
5. (a) $5(x+2) > \frac{8x-7}{3}$		
$15(x+2) > 8x-7$		
$15x+30 > 8x-7$		
$7x > -37$	1M	給將 x 放在一邊
$x > -\frac{37}{7}$	1A	$x > -5\frac{2}{7}$
$6-x \geq 8$		
$x \leq -2$		
因此，所求的範圍為 $-\frac{37}{7} < x \leq -2$ 。	1A	$-5\frac{2}{7} < x \leq -2$
(b) $-5, -4, -3, -2$	1A	
	---------(4)	
6. (a) A' 的坐標為 $(6,4)$ 。	1A	接受 $A'(6,4)$ 或 $A'=(6,4)$
B' 的坐標為 $(-3,-2)$ 。	1A	
(b) $A'O$ 的斜率 $=\frac{4-0}{6-0}=\frac{2}{3}$	1M	任何一項
$B'O$ 的斜率 $=\frac{-2-0}{-3-0}=\frac{2}{3}$		接受 $m_{A'O}=\frac{2}{3}$
$\because$ $A'O$ 的斜率 $=B'O$ 的斜率，		
又 O 為公共點，		
$\therefore$ $A'OB'$ 成一直線。	1	必須顯示理由
$A'O=\sqrt{(6-0)^2+(4-0)^2}=2\sqrt{13}$	1M	給任何一項
$B'O=\sqrt{(0+3)^2+(0+2)^2}=\sqrt{13}$		
$A'B'=\sqrt{(6+3)^2+(4+2)^2}=3\sqrt{13}$		
$\because$ $A'B'=A'O+B'O$		
$\therefore$ $A'OB'$ 成一直線。	1	必須顯示理由
	---------(4)	
7. 由已知概率得 $\frac{a}{b}=\frac{3}{5}$	1M	
$5a=3b$ ……(*)		
又 $a-9=b-17$	1M	
$a=b-8$		
代入 (*) ，得 $5(b-8)=3b$		
$b=20$	1A	
$a=12$	1A	
	---------(4)	

解	分	備註
8. 連 AD。		
$\angle ACD = 180° - \theta$	1A	
$\angle ADC = 180° - \angle ABC$		
$\angle CAD = \frac{1}{2} \times \angle COD$	1M	
$= \frac{1}{2} \times 64°$		
$= 32°$	1A	
在 ΔACD 中，		
$32° + 180° - \theta + 180° - \angle ABC = 180°$	1M	
$\angle ABC = 212° - \theta$	1A	
連 AD。		
$\angle ACD = 180° - \theta$	1A	
$\because$ $OC = OD$		
$\therefore$ $\angle OCD = \frac{180° - 64°}{2}$	1M	
$= 58°$		
$\angle OCA = \angle OAC$		
$= 180° - \theta - 58°$		
$= 122° - \theta$	1A	
$\angle AOC = 180° - 2(122° - \theta)$		
$= 2\theta - 64°$		
反角 $\angle AOC = 360° - (2\theta - 64°)$		
$= 424° - 2\theta$		
$\angle ABC = \frac{1}{2} \times$ 反角 $\angle AOC$	1M	
$= \frac{1}{2}(424° - 2\theta)$		
$= 212° - \theta$	1A	
	---------(5)	
9. (a) $C = as + bs^2$，其中 a、b 為非零的常數。	1A	
代入 $s = 4$，$C = 20$ 及 $s = 6$，$C = 36$，得		
$20 = 4a + 16b$	1M	給任何一項
$a + 4b = 5$ ……(1)		
$36 = 6a + 36b$		
$a + 6b = 6$ ……(2)		
解 (1)、(2) 兩式，得 $a = 3$，$b = \frac{1}{2}$。	1A	給兩項正確
$\therefore$ $C = 3s + \frac{1}{2}s^2$		
(b) $3s + \frac{1}{2}s^2 = 45.5$		
$s^2 + 6s - 91 = 0$	1M	
$(s + 13)(s - 7) = 0$		
$\therefore$ $s = -13$ (捨) 或 $s = 7$	1A	給兩項正確
所求周界為 7 m。		
	---------(5)	

解	分	備註
10. (a) $\frac{426+20+a}{18}=25$	1M	
$a=4$	1A	
設該三名新球員的平均年齡為 n 歲，		
則 $\frac{25\times18-33-33+3n}{19}=24$	1M	
$n=24$	1A	
故該三名新球員的平均年齡為 24 歲。		
	---------(4)	
(b) 由於該三名新球員的平均年齡為 24 歲。		
故有以下的四種情況：		
(1) 2 個數據小於 24， 1 個數據大於 24，則 $m=23$；	1M	考慮至少二種情況
(2) 1 個數據小於 24， 1 個數據等於 24，又 1 個數據大於 24，則 $m=24$；		
(3) 1 個數據小於 24， 2 個數據大於 24，則 $m=24$；		
(4) 3 個數據都等於 24，則 $m=24$。		
∴ m 的可取值為 23 及 24。	1A	
	---------(2)	
11. (a) $f(x)=(x^2-2x-3)(4x+5)+6x+k$		
$f(2)=(4-4-3)(8+5)+12+k=-21$	1M	
$k=6$	1A	
	---------(2)	
(b) $f(x)=0$		
$(x^2-2x-3)(4x+5)+6x+6=0$		
$(x-3)(x+1)(4x+5)+6(x+1)=0$		
$(x+1)[(x-3)(4x+5)+6]=0$	1M	
$(x+1)(4x^2-7x-9)=0$	1A	
$4x^3-3x^2-16x-9=0$ $(x+1)(4x^2-7x-9)=0$	1M+1A	
$x=-1$ 或 $x=\frac{7\pm\sqrt{193}}{8}$ (不是有理數)	1A	
因此，不同意該宣稱。	1A	必須顯示理由
	---------(4)	

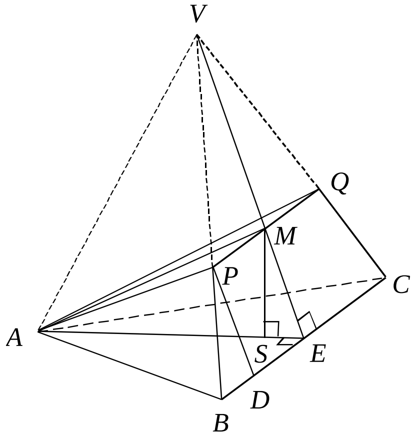
解	分	備註
12. (a) $\because$ $AB = BC$ (正方形的邊)		或正方形的性質
$BE = CF$ (已知)		
$\therefore$ $AB + BE = BC + CF$		
即 $AE = BF$		
又 $AD = BA$ (正方形的邊)		或正方形的性質
$\angle DAE = \angle ABF$ (正方形的角)		或正方形的性質
$\therefore$ $\Delta ADE \cong \Delta BAF$ (SAS)		
評分標準：		
情況 1 附有正確理由的任何正確證明。	2	
情況 2 未附有正確理由的任何正確證明。	1	
	---------(2)	
(b) (i) $AE = 6 + 2 = 8$		
ΔADE 的面積 $= \dfrac{8 \times 6}{2} = 24\ \text{cm}^2$	1A	
(ii) 作 $AN \perp DE$ 使得垂足為 N。		
則 AN 為 A 至 DE 的最短距離。	1M	
$DE = \sqrt{8^2 + 6^2} = 10$		
$\dfrac{10 \times AN}{2} = 24$	1M	
$AN = 4.8$	1A	
即 A 至 DE 的最短距離為 4.8 cm。		
因此 DE 上不存在一點 K 使得 A 與 K 的距離少於 4.8 cm。	1A	必須顯示理由
	---------(5)	

解	分	備註
13. (a) ∵ C_1 的圖像與正 x 軸相切		
∴ $\Delta = k^2 - 144 = 0$	1M	
$k = 12$ (捨) 或 $k = -12$	1A	
	---------(2)	
(b) (i) M 點的坐標為 $(6,0)$ 。		
R 點的 x 坐標為 3。		兩項全對
代入 C_1，得		
R 點的 y 坐標為 9。	1A	
$C_2 : y = p(x-3)^2 + 9$	1M	
代入點 $(6,0)$ ，得		或代入點 $(0,0)$
$0 = 9p + 9$		
$p = -1$		
∴ $p = -1$ ， $q = 6$ ， $r = 0$	1A	三項全對
$C_2 : y = px(x-6)$	1M	接受 $y = kx(x-6)$
代入 $(3,9)$ ，得		
$9 = -9p$		
$p = -1$		
$C_2 : y = -x(x-6)$		
$y = -x^2 + 6x$		
∴ $p = -1$ ， $q = 6$ ， $r = 0$	1A	三項全對
(ii) N 點的坐標為 $(0,36)$ 。		
ΔMNO 的面積 $= \frac{36 \times 6}{2} = 108$	1A	
由 R 作 $RS \perp OM$ 使垂足為 S 。		
$RS = 9$ ， $OS = SM = 3$		
ΔMNR 的面積		
$= 108 - \frac{(36+9) \times 3}{2} - \frac{9 \times 3}{2}$	1M	
$= 27$		
$= \frac{1}{4} \times 108$		
$= \Delta MNO$ 面積的 $\frac{1}{4}$		
因此，我同意該宣稱。	1A	必須顯示理由
	------(6)	

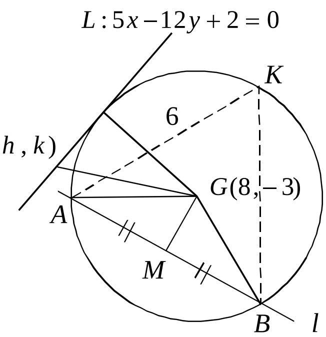
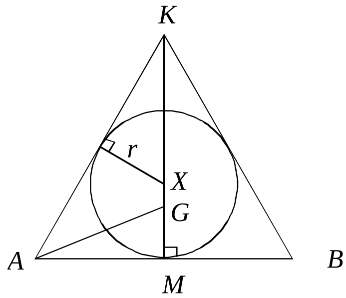
解	分	備註
14. (a) 該容器的曲面面積		
$=\pi\times10\times\sqrt{10^2+24^2}$	1M	接受 $\pi\times10\times26$
$=260\pi\ (\text{cm}^2)$	1A	
被水所浸濕的曲面面積		
$=260\pi\times(\sqrt[3]{\frac{64}{125}})^2$	1M	
$=260\pi\times\frac{16}{25}$		
$=\frac{832}{5}\pi\ (\text{cm}^2)$	1A	接受 $166\frac{2}{5}\pi$ 或 166.4π
水面的半徑		
$=10\times\sqrt[3]{\frac{64}{125}}$	1M	給任何一項（水面的半徑／水的深度）
$=8$		
水的深度		
$=24\times\sqrt[3]{\frac{64}{125}}$		
$=\frac{96}{5}$		
被水所浸濕的曲面面積		
$=\pi\times8\times\sqrt{8^2+(\frac{96}{5})^2}$	1M+1A	
$=\pi\times8\times\frac{104}{5}$		
$=\frac{832}{5}\pi\ (\text{cm}^2)$	1A	
	---------(4)	
(b) 設容器內水的深度為 h cm，		
則 $\frac{260\pi-\frac{832}{5}\pi}{260\pi}=(\frac{24-h}{24})^2$	1M+1A	
$\frac{468}{1300}=(\frac{24-h}{24})^2$		
$\frac{9}{25}=(\frac{24-h}{24})^2$		
$\frac{24-h}{24}=\frac{3}{5}$	1A	
$h=9.6$		
<14.8		
因此，我不同意該宣稱。	1A	必須顯示理由
	---------(4)	

解	分	備註
15. (a) 可排成六位數的個數		
$=P_1^5\cdot P_5^5$		接受 $C_1^5\cdot P_5^5$
$=600$	1A	
	---------(1)	
(b) 可排成六位偶數的個數		
$=P_5^5+P_1^3\cdot P_1^4\cdot P_4^4$	1M	接受 $P_5^5+C_1^3\cdot C_1^4\cdot P_4^4$
$=408$	1A	
可排成六位偶數的個數		
$=600-P_1^2\cdot P_1^4\cdot P_4^4$	1M	接受 $600-C_1^2\cdot C_1^4\cdot P_4^4$
$=408$	1A	
	---------(2)	
16. (a) 設 a 及 r 分別為數列的首項及公比，		
則 $ar=600$ ……(1)		
又 $\dfrac{a}{1-r}=3200$ ……(2)		
$\dfrac{(1)}{(2)}$ 得 $r(1-r)=\dfrac{3}{16}$	1M	
$16r^2-16r+3=0$		
$(4r-1)(4r-3)=0$		
$r=\dfrac{1}{4}$ 或 $r=\dfrac{3}{4}$		
當 $r=\dfrac{1}{4}$ 時，$a=2400$ (捨)		
當 $r=\dfrac{3}{4}$ 時，$a=800$		
因此，首項為 800。	1A	
	---------(2)	
(b) $800(\dfrac{3}{4})^n+800(\dfrac{3}{4})^{2n}>100$	1M	
$8(0.75^n)^2+8(0.75^n)-1>0$		
$0.75^n<\dfrac{-8-\sqrt{96}}{16}$ (捨) 或 $0.75^n>\dfrac{-8+\sqrt{96}}{16}$		
$\log(0.75^n)>\log\dfrac{-8+\sqrt{96}}{16}$	1M	
$n\log 0.75>\log\dfrac{-8+\sqrt{96}}{16}$		
$n<7.598445697$		
由於 n 為一正整數，		
$\therefore$ n 的最大值為 7。	1A	
	---------(3)	

解	分	備註
17. (a) (i) $L_3: x+y=30$	1A	
(ii) $x+y\geq 5$ 及 $x+y\leq 30$	1A ---------(2)	兩項全對
(b) (i) B 工場分配蛋糕給丙的數量 $=25-(30-x-y)$ $=x+y-5$	1A	
B 工場分配蛋糕給丙的數量 $=60-(20-x)-(45-y)$ $=x+y-5$	1A	
(ii) $\begin{cases}0\leq x\leq 20\\0\leq y\leq 30\\0\leq x+y-5\leq 25\end{cases}$	1A	($y\leq 30$ 可略去)
即 $\begin{cases}0\leq x\leq 20\\0\leq y\leq 30\\5\leq x+y\leq 30\end{cases}$		
總運費 $\mathrm{f}(x,y)=\$5[8x+4y+30-x-y+2(20-x)+(45-y)+x+y-5]$ $=\$(30x+15y+550)$	1M	
$\mathrm{f}(0,5)=625$ ； $\mathrm{f}(0,30)=1000$ ； $\mathrm{f}(20,10)=1300$ ； $\mathrm{f}(20,0)=1150$ ； $\mathrm{f}(5,0)=700$	1M	
因此，總運費的最小值為 \$625。	1A ---------(5)	

解	分	備註
18. (a) $\angle APB = 180° - 78° - 60°$		
$= 42°$		
在 ΔAPB 中，		
$\dfrac{12}{\sin 42°} = \dfrac{PB}{\sin 60°}$	1M	
$PB \approx 15.53105589$		
≈ 15.5 (cm)	1A	接受答案準確至 15.5 cm
	--------(2)	
(b) (i) 		
設 PQ 的中點 M。		
作 $PD \perp BC$ 及 $ME \perp BC$ 使垂足分別為		
D 及 E。並連 AE。		
$ME = PD$	1M	
$= PB \sin 78°$		
$\approx 15.53105589 \sin 78°$		
≈ 15.19166506		
E 為 BC 的中點		
$BE = 6$		
$VE = 6 \tan 78°$	1M	給任何一項
≈ 28.22778066		
$VA = VB$		
$= \dfrac{6}{\cos 78°}$		
≈ 28.85840607		
又在 rt.ΔABE 中，		
$AE = \sqrt{12^2 - 6^2}$		
$= \sqrt{108}$		
在 ΔAVE 中，		
$\cos \angle VEA \approx \dfrac{28.22778066^2 + 108 - 28.85840607^2}{2(28.22778066)(\sqrt{108})}$	1M	
$\angle VEA \approx 82.95091613°$		
由 M 作 $MS \perp AE$ 使垂足為 E，並連 AM。		
$\alpha = \angle MAS$	1M	指出所求的夾角
$MS = ME \sin \angle VEA$		
$\approx 15.19166506 \sin 82.95091613°$		
≈ 15.07683711		

解	分	備註
$AS = AE - SE$		
$\approx \sqrt{108} - 15.19166506\cos 82.95091613°$		
≈ 8.527989965		
在 rt.ΔMAS 中，		
$\tan\alpha = \dfrac{MS}{AS}$		
$\approx \dfrac{15.07683711}{8.527989965}$		
$\alpha \approx 60.50597106°$		
$\approx 60.5°$	1A	接受答案準確至 60.5°
(ii) $\because$ $AM \perp PQ$		
$\therefore$ AM 是斜面 APQ 上的最大斜率的直線。		
$\therefore$ α 是最大的傾角。	1M	
即 $\alpha > \beta$ 。		$\beta \approx 59.3°$
因此，不同意該宣稱。	1A	必須顯示理由。
	--------(7)	

解	分	備註
19. (a) $GP^2=(h-8)^2+(k+3)^2$		
$=(\frac{12k-2}{5}-8)^2+(k+3)^2$	1M	
$=\frac{144k^2-1008k+1764}{25}+k^2+6k+9$		
$=\frac{169}{25}k^2-\frac{858}{25}k+\frac{1989}{25}$	1A	
$=\frac{169}{25}(k^2-\frac{66}{33}k)+\frac{1989}{25}$		
$=\frac{169}{25}(k^2-\frac{66}{13}k+\frac{1089}{169}-\frac{1089}{169})+\frac{1989}{25}$	1M	
$=\frac{169}{25}(k-\frac{33}{13})^2+36$		
當 $k=\frac{33}{13}$ 時，GP^2 有最小值 36。	1A	
	---------(4)	
(b) (i) 圓 C 的方程為		
$(x-8)^2+(y+3)^2=36$	1A	或 $x^2+y^2-16x+6y+37=0$
(ii) 設 M 為 AB 的中點，		
則 $GM\perp AB$。		
$GA=6$，$AM=4\sqrt{2}$		
$\sin\angle AGM=\frac{4\sqrt{2}}{6}$		
$\angle AGM=70.52877937°$		
$\angle AKB=\frac{2\times\angle AGM}{2}$		
$=\angle AGM$		
$\approx 70.52877937°$		
$\approx 70.5°$	1A	接受答案準確至 70.5°
或 $\angle AKB\approx 180°-70.52877937°$		
$\approx 109.4712206°$		
$\approx 109°$	1A	接受答案準確至 109°
當 ΔABK 為一銳角等腰三角形時，		
KGM 成一直線，		
且 $KGM\perp AB$		
$GM=\sqrt{6^2-(4\sqrt{2})^2}$	1M	
$=2$		
$KM=6+2=8$	1A	

解	分	備註
設內切圓的圓心為 X，半徑為 r。 $\angle AKX = \frac{1}{2}\angle AKB$ $\approx \frac{1}{2}\times 70.52877937°$ $\approx 35.26438969°$ $\because\ KM = KX + XM = 8$ $\therefore\ \frac{r}{\sin 35.26438969°} + r \approx 8$ $r \approx \frac{8}{1+\frac{1}{\sin 35.26438969°}}$ ≈ 2.928203231 ≈ 2.93 內切圓的半徑為 2.93 。	 1M 1A --------(7)	 接受答案準確至 2.93

Paper 1

Solution	Marks	Remarks
1. $\frac{a+b}{2}=\frac{4b-1}{3}$		
$3(a+b)=2(4b-1)$ | $\frac{3}{2}(a+b)=4b-1$	1M	
$3a+3b=8b-2$ | $\frac{3}{2}a+\frac{3}{2}b=4b-1$		
$3a+2=5b$ | $\frac{3}{2}a+1=\frac{5}{2}b$	1M	for putting b on one side
$b=\frac{3a+2}{5}$ | $b=\frac{3a+2}{5}$	1A	
	---------(3)	
2. $\frac{x^2y^{-3}}{(x^3y^{-1})^6}$		
$=\frac{x^2y^{-3}}{x^{18}y^{-6}}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
$=\frac{y^{-3+6}}{x^{18-2}}$	1M	for $c^{-p}=\frac{1}{c^p}$ or $\frac{c^p}{c^q}=c^{p-q}$
$=\frac{y^3}{x^{16}}$	1A	
	---------(3)	
3. (a) 38.2	1A	
(b) Percentage error		
$=\frac{38.26-38.2}{38.26}\times 100\%$	1M	
$\approx 0.157\%$	1A	
	---------(3)	
4. (a) $8m^3-4m^2n$		
$=4m^2(2m-n)$	1A	
(b) $8m^3-4m^2n-18mn^2+9n^3$		
$=4m^2(2m-n)-18mn^2+9n^3$	1M	for using the result of (a)
$=4m^2(2m-n)-9n^2(2m-n)$		
$=(2m-n)(4m^2-9n^2)$	1M	
$=(2m-n)(2m+3n)(2m-3n)$	1A	or equivalent
	---------(4)	

Solution	Marks	Remarks
5. (a) $5(x+2) > \frac{8x-7}{3}$		
$15(x+2) > 8x-7$		
$15x+30 > 8x-7$		
$7x > -37$	1M	for putting x on one side
$x > -\frac{37}{7}$	1A	$x > -5\frac{2}{7}$
$6-x \geq 8$		
$x \leq -2$		
Thus, the required range is $-\frac{37}{7} < x \leq -2$.	1A	$-5\frac{2}{7} < x \leq -2$
(b) $-5, -4, -3, -2$	1A	
	---------(4)	
6. (a) The coordinates of A' are $(6, 4)$.	1A	accept $A'(6, 4)$ or $A' = (6, 4)$
The coordinates of B' are $(-3, -2)$.	1A	
(b) The slope of $A'O = \frac{4-0}{6-0} = \frac{2}{3}$	1M	either one
The slope of $B'O = \frac{-2-0}{-3-0} = \frac{2}{3}$		accept $m_{A'O} = \frac{2}{3}$
$\because$ The slope of $A'O$ = the slope of $B'O$		
Also, O is a common point .		
$\therefore$ $A'OB'$ is a straight line .	1	f.t.
$A'O = \sqrt{(6-0)^2 + (4-0)^2} = 2\sqrt{13}$	1M	either one
$B'O = \sqrt{(0+3)^2 + (0+2)^2} = \sqrt{13}$		
$A'B' = \sqrt{(6+3)^2 + (4+2)^2} = 3\sqrt{13}$		
$\because$ $A'B' = A'O + B'O$		
$\therefore$ $A'OB'$ is a straight line .	1	f.t.
	---------(4)	
7. From the given probability , we get $\frac{a}{b} = \frac{3}{5}$	1M	
$5a = 3b$ ……(*)		
Also , $a-9 = b-17$	1M	
$a = b-8$		
Sub. into (*) , we have $5(b-8) = 3b$		
$b = 20$	1A	
$a = 12$	1A	
	---------(4)	

	Solution	Marks	Remarks
8.	Join AD .		
	$\angle ACD = 180° - \theta$	1A	
	$\angle ADC = 180° - \angle ABC$		
	$\angle CAD = \frac{1}{2} \times \angle COD$	1M	
	$= \frac{1}{2} \times 64°$		
	$= 32°$	1A	
	In ΔACD ,		
	$32° + 180° - \theta + 180° - \angle ABC = 180°$	1M	
	$\angle ABC = 212° - \theta$	1A	
	Join AD .		
	$\angle ACD = 180° - \theta$	1A	
	$\because \ OC = OD$		
	$\therefore \ \angle OCD = \frac{180° - 64°}{2}$	1M	
	$= 58°$		
	$\angle OCA = \angle OAC$		
	$= 180° - \theta - 58°$		
	$= 122° - \theta$	1A	
	$\angle AOC = 180° - 2(122° - \theta)$		
	$= 2\theta - 64°$		
	reflex $\angle AOC = 360° - (2\theta - 64°)$		
	$= 424° - 2\theta$		
	$\angle ABC = \frac{1}{2} \times$ reflex $\angle AOC$	1M	
	$= \frac{1}{2}(424° - 2\theta)$		
	$= 212° - \theta$	1A	
		---------(5)	
9.	(a) $C = as + bs^2$, where a , b are non-zero constants .	1A	
	Sub. $s = 4$, $C = 20$ and $s = 6$, $C = 36$, we have		
	$20 = 4a + 16b$	1M	either one
	$a + 4b = 5$ ……(1)		
	$36 = 6a + 36b$		
	$a + 6b = 6$ ……(2)		
	Solving (1) and (2) , we have $a = 3$ and $b = \frac{1}{2}$.	1A	for both correct
	$\therefore \ C = 3s + \frac{1}{2}s^2$		
	(b) $3s + \frac{1}{2}s^2 = 45.5$		
	$s^2 + 6s - 91 = 0$	1M	
	$(s + 13)(s - 7) = 0$		
	$\therefore \ s = -13$ (rejected) or $s = 7$	1A	for both correct
	Thus , the perimeter of the advertising board is 7 m .		
		---------(5)	

	Solution	Marks	Remarks
10. (a)	$\frac{426+20+a}{18}=25$	1M	
	$a=4$	1A	
	Let n be the mean age of the three new players ,		
	then $\frac{25\times 18-33-33+3n}{19}=24$	1M	
	$n=24$	1A	
	Thus , the mean age of the three new players is 24 .		
		---------(4)	
(b)	As the mean age of the three new players is 24 ,		
	there are four cases :		
	(1) 2 data are less than 24 and 1 datum is greater than 24 , then $m=23$;	1M	consider at least two cases
	(2) 1 datum is less than 24 , 1 datum is equal to 24 and 1 datum is greater than 24 , then $m=24$;		
	(3) 1 datum is less than 24 and 2 data are greater than 24 , then $m=24$;		
	(4) 3 data are equal to 24 , then $m=24$.		
	$\therefore$ The possible values of m are 23 and 24 .	1A	
		---------(2)	
11. (a)	$f(x)=(x^2-2x-3)(4x+5)+6x+k$		
	$f(2)=(4-4-3)(8+5)+12+k=-21$	1M	
	$k=6$	1A	
		---------(2)	
(b)	$f(x)=0$		
	$(x^2-2x-3)(4x+5)+6x+6=0$		
	$(x-3)(x+1)(4x+5)+6(x+1)=0$		
	$(x+1)[(x-3)(4x+5)+6]=0$	1M	
	$(x+1)(4x^2-7x-9)=0$	1A	
	$4x^3-3x^2-16x-9=0$		
	$(x+1)(4x^2-7x-9)=0$	1M+1A	
	$x=-1$ or $x=\frac{7\pm\sqrt{193}}{8}$ (which are not rational numbers)	1A	
	Thus , the claim is disagreed .	1A	f.t.
		---------(4)	

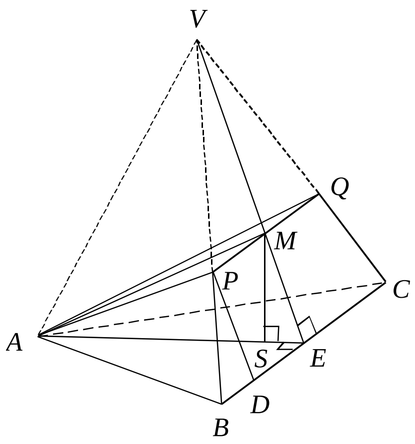
		Solution	Marks	Remarks
12.	(a)	$\because$ $AB = BC$ (sides of a square)		property of square
		$BE = CF$ (given)		
		$\therefore$ $AB + BE = BC + CF$		
		i.e. $AE = BF$		
		And $AD = BA$ (sides of a square)		property of square
		$\angle DAE = \angle ABF$ (angles of a square)		property of square
		$\therefore$ $\Delta ADE \cong \Delta BAF$ (SAS)		
		Marking Scheme :		
		Case 1 Any correct proof with correct reasons .	2	
		Case 2 Any correct proof without reasons .	1	
			---------(2)	
	(b)	(i) $AE = 6 + 2 = 8$		
		The area of $\Delta ADE = \frac{8 \times 6}{2} = 24$ cm^2	1A	
		(ii) Construct $AN \perp DE$ such that N is the foot of the perpendicular .		
		Then AN is the shortest distance from A to DE .	1M	
		$DE = \sqrt{8^2 + 6^2} = 10$		
		$\frac{10 \times AN}{2} = 24$	1M	
		$AN = 4.8$	1A	
		The shortest distance from A to DE is 4.8 cm . Therefore , there does not exist a point K lying on DE such that the distance between A and K is less than 4.8 cm .	1A	f.t.
			---------(5)	

		Solution	Marks	Remarks
13.	(a)	$\because$ The graph of C_1 touches the positive x-axis .		
		$\therefore$ $\Delta = k^2 - 144 = 0$	1M	
		$k = 12$ (rejected) or $k = -12$	1A	
			---------(2)	
	(b)	(i) The coordinates of the point M are $(6, 0)$.		
		The x-coordinate of the point R is 3 .		for both correct
		Sub. into C_1 ,		
		the y-coordinate of the point R is 9 .	1A	
		$C_2 : y = p(x-3)^2 + 9$	1M	
		sub. $(6, 0)$, we have		or sub. $(0, 0)$
		$0 = 9p + 9$		
		$p = -1$		
		$\therefore$ $p = -1$, $q = 6$, $r = 0$	1A	for all correct
		$C_2 : y = px(x-6)$	1M	accept $y = kx(x-6)$
		Sub. $(3, 9)$, we have		
		$9 = -9p$		
		$p = -1$		
		$C_2 : y = -x(x-6)$		
		$y = -x^2 + 6x$		
		$\therefore$ $p = -1$, $q = 6$, $r = 0$	1A	for all correct
		(ii) The coordinates of the point N are $(0, 36)$.		
		The area of ΔMNO $= \frac{36 \times 6}{2} = 108$	1A	
		Construct $RS \perp OM$ such that S is the foot of the perpendicular.		
		$RS = 9$, $OS = SM = 3$		
		The area of ΔMNR		
		$= 108 - \frac{(36+9) \times 3}{2} - \frac{9 \times 3}{2}$	1M	
		$= 27$		
		$= \frac{1}{4} \times 108$		
		$= \frac{1}{4} \times$ the area of ΔMNO		
		Thus , the claim is agreed .	1A	f.t.
			------(6)	

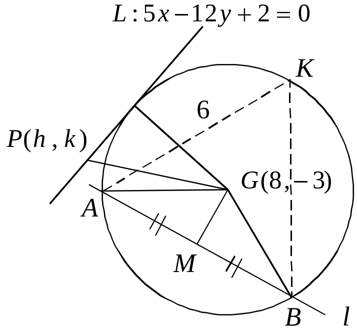
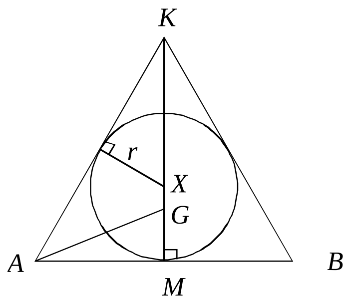
	Solution	Marks	Remarks
14. (a)	The curved surface area of the vessel		
	$=\pi\times10\times\sqrt{10^2+24^2}$	1M	accept $\pi\times10\times26$
	$=260\pi\ (\text{cm}^2)$	1A	
	The area of the wet curved surface of the vessel		
	$=260\pi\times(\sqrt[3]{\frac{64}{125}})^2$	1M	
	$=260\pi\times\frac{16}{25}$		
	$=\frac{832}{5}\pi\ (\text{cm}^2)$	1A	accept $166\frac{2}{5}\pi$ or 166.4π
	The radius of water surface		
	$=10\times\sqrt[3]{\frac{64}{125}}$	1M	either one
	$=8$		
	The depth of water		
	$=24\times\sqrt[3]{\frac{64}{125}}$		
	$=\frac{96}{5}$		
	The area of the wet curved surface of the vessel		
	$=\pi\times8\times\sqrt{8^2+(\frac{96}{5})^2}$	1M+1A	
	$=\pi\times8\times\frac{104}{5}$		
	$=\frac{832}{5}\pi\ (\text{cm}^2)$	1A	
		---------(4)	
(b)	Let h cm be the depth of water in the vessel .		
	Then $\frac{260\pi-\frac{832}{5}\pi}{260\pi}=(\frac{24-h}{24})^2$	1M+1A	
	$\frac{468}{1300}=(\frac{24-h}{24})^2$		
	$\frac{9}{25}=(\frac{24-h}{24})^2$		
	$\frac{24-h}{24}=\frac{3}{5}$	1A	
	$h=9.6$		
	<14.8		
	Thus , the claim is disagreed .	1A	f.t.
		---------(4)	

Solution	Marks	Remarks
15. (a) The required number		
$= P_1^5 \cdot P_5^5$		accept $C_1^5 \cdot P_5^5$
$= 600$	1A	
	---------(1)	
(b) The required number		
$= P_5^5 + P_1^3 \cdot P_1^4 \cdot P_4^4$	1M	accept $P_5^5 + C_1^3 \cdot C_1^4 \cdot P_4^4$
$= 408$	1A	
The required number		
$= 600 - P_1^2 \cdot P_1^4 \cdot P_4^4$	1M	accept $600 - C_1^2 \cdot C_1^4 \cdot P_4^4$
$= 408$	1A	
	---------(2)	
16. (a) Let a and r be the first term and the common ratio of the sequence respectively .		
Then $ar = 600$ $\cdots\cdots(1)$		
Also, $\dfrac{a}{1-r} = 3200$ $\cdots\cdots(2)$		
$\dfrac{(1)}{(2)}$, we get $r(1-r) = \dfrac{3}{16}$	1M	
$16r^2 - 16r + 3 = 0$		
$(4r-1)(4r-3) = 0$		
$r = \dfrac{1}{4}$ or $r = \dfrac{3}{4}$		
When $r = \dfrac{1}{4}$, $a = 2400$ (rejected)		
When $r = \dfrac{3}{4}$, $a = 800$		
Thus , the first term is 800 .	1A	
	---------(2)	
(b) $800(\dfrac{3}{4})^n + 800(\dfrac{3}{4})^{2n} > 100$	1M	
$8(0.75^n)^2 + 8(0.75^n) - 1 > 0$		
$0.75^n < \dfrac{-8-\sqrt{96}}{16}$ (rejected) or $0.75^n > \dfrac{-8+\sqrt{96}}{16}$		
$\log(0.75^n) > \log\dfrac{-8+\sqrt{96}}{16}$	1M	
$n\log 0.75 > \log\dfrac{-8+\sqrt{96}}{16}$		
$n < 7.598445697$		
Since n is a positive integer .		
$\therefore$ The greatest value of n is 7 .	1A	
	---------(3)	

	Solution	Marks	Remarks
17. (a)	(i) $L_3 : x + y = 30$	1A	
	(ii) $x + y \geq 5$ and $x + y \leq 30$	1A	for both correct
		---------(2)	
(b)	(i) The number of cakes allocated to Cindy from workshop B $= 25 - (30 - x - y)$ $= x + y - 5$	1A	
	The number of cakes allocated to Cindy from workshop B $= 60 - (20 - x) - (45 - y)$ $= x + y - 5$	1A	
	(ii) $\begin{cases} 0 \leq x \leq 20 \\ 0 \leq y \leq 30 \\ 0 \leq x + y - 5 \leq 25 \end{cases}$	1A	($y \leq 30$ can be omitted)
	i.e. $\begin{cases} 0 \leq x \leq 20 \\ 0 \leq y \leq 30 \\ 5 \leq x + y \leq 30 \end{cases}$		
	The total transportation charge $f(x, y) = \$5[8x + 4y + 30 - x - y + 2(20 - x) + (45 - y) + x + y - 5]$ $= \$(30x + 15y + 550)$	1M	
	$f(0, 5) = 625$; $f(0, 30) = 1000$; $f(20, 10) = 1300$; $f(20, 0) = 1150$; $f(5, 0) = 700$	1M	
	Thus, the least value of the total transportation charge is \$625 .	1A	
		---------(5)	

Solution	Marks	Remarks
18. (a) $\angle APB = 180° - 78° - 60°$ $= 42°$ In ΔAPB , $\frac{12}{\sin 42°} = \frac{PB}{\sin 60°}$	1M	
$PB \approx 15.53105589$ ≈ 15.5 (cm)	1A --------(2)	r.t. 15.5 cm
(b) (i) 		
Let M be the mid-point of PQ . Construct $PD \perp BC$ and $ME \perp BC$ such that D and E are the feet of the perpendicular . Join AE . $ME = PD$ $= PB \sin 78°$ $\approx 15.53105589 \sin 78°$ ≈ 15.19166506	1M	
E is the mid-point of BC . $BE = 6$ $VE = 6 \tan 78°$ ≈ 28.22778066 $VA = VB$ $= \frac{6}{\cos 78°}$ ≈ 28.85840607	1M	either one
In rt.ΔABE , $AE = \sqrt{12^2 - 6^2}$ $= \sqrt{108}$ In ΔAVE , $\cos \angle VEA \approx \frac{28.22778066^2 + 108 - 28.85840607^2}{2(28.22778066)(\sqrt{108})}$ $\angle VEA \approx 82.95091613°$	1M	
Construct $MS \perp AE$ such that E is the foot of the perpendicular . Join AM . $\alpha = \angle MAS$	1M	for identifying the required angle
$MS = ME \sin \angle VEA$ $\approx 15.19166506 \sin 82.95091613°$ ≈ 15.07683711		

Solution	Marks	Remarks
$AS = AE - SE$		
$\approx \sqrt{108} - 15.19166506\cos 82.95091613°$		
≈ 8.527989965		
In rt.ΔMAS ,		
$\tan\alpha = \dfrac{MS}{AS}$		
$\approx \dfrac{15.07683711}{8.527989965}$		
$\alpha \approx 60.50597106°$		
$\approx 60.5°$	1A	r.t. 60.5°
(ii) $\because$ $AM \perp PQ$		
$\therefore$ AM is the line of greatest slope on the plane APQ .		
$\therefore$ α is the greatest inclination .	1M	
That is, $\alpha > \beta$.		$\beta \approx 59.3°$
Thus , the claim is disagreed .	1A	f.t.
	--------(7)	

Solution	Marks	Remarks
19. (a) $GP^2 = (h-8)^2 + (k+3)^2$		
$= (\frac{12k-2}{5} - 8)^2 + (k+3)^2$	1M	
$= \frac{144k^2 - 1008k + 1764}{25} + k^2 + 6k + 9$		
$= \frac{169}{25}k^2 - \frac{858}{25}k + \frac{1989}{25}$	1A	
$= \frac{169}{25}(k^2 - \frac{66}{33}k) + \frac{1989}{25}$		
$= \frac{169}{25}(k^2 - \frac{66}{13}k + \frac{1089}{169} - \frac{1089}{169}) + \frac{1989}{25}$	1M	
$= \frac{169}{25}(k - \frac{33}{13})^2 + 36$		
When $k = \frac{33}{13}$, the minimum value of GP^2 is 36 .	1A	
	---------(4)	
(b) (i) The equation of the circle C is $(x-8)^2 + (y+3)^2 = 36$	1A	or $x^2 + y^2 - 16x + 6y + 37 = 0$
(ii) Let M be the mid-point of AB . Then $GM \perp AB$.		
$GA = 6$, $AM = 4\sqrt{2}$		
$\sin\angle AGM = \frac{4\sqrt{2}}{6}$		
$\angle AGM = 70.52877937°$		
$\angle AKB = \frac{2 \times \angle AGM}{2}$		
$= \angle AGM$		
$\approx 70.52877937°$		
$\approx 70.5°$	1A	r.t. $70.5°$
or $\angle AKB \approx 180° - 70.52877937°$		
$\approx 109.4712206°$		
$\approx 109°$	1A	r.t. $109°$
When ΔABK is an acute-angled isosceles triangle , KGM is a straight line and $KGM \perp AB$.		
$GM = \sqrt{6^2 - (4\sqrt{2})^2}$	1M	
$= 2$		
$KM = 6 + 2 = 8$	1A	

Solution	Marks	Remarks
Let X be the centre and r be the radius of the inscribed circle .		
$\angle AKX = \frac{1}{2}\angle AKB$		
$\approx \frac{1}{2} \times 70.52877937°$		
$\approx 35.26438969°$		
$\because \quad KM = KX + XM = 8$		
$\therefore \quad \frac{r}{\sin 35.26438969°} + r \approx 8$	1M	
$r \approx \frac{8}{1 + \frac{1}{\sin 35.26438969°}}$		
≈ 2.928203231		
≈ 2.93	1A	r.t. 2.93
The radius of the inscribed circle is 2.93 .	--------(7)	

HYC Mock Examination 2018/19 MATHEMATICS Compulsory Part PAPER 2

1. D
2. C
3. A
4. A
5. D
6. D
7. B
8. C
9. C
10. C
11. D
12. B
13. D
14. A
15. C
16. B
17. B
18. A
19. C
20. B
21. B
22. A
23. A
24. D
25. C
26. D
27. C
28. A
29. B
30. D
31. B
32. A
33. C
34. B
35. B
36. A
37. C
38. D
39. D
40. A
41. A
42. B
43. C
44. A
45. D