Paper 1

	Solution	Marks	Remarks
1.	$\dfrac{a+b}{2}=\dfrac{4b-1}{3}$		
	$3(a+b)=2(4b-1)$	$\dfrac{3}{2}(a+b)=4b-1$	1M	
	$3a+3b=8b-2$	$\dfrac{3}{2}a+\dfrac{3}{2}b=4b-1$		
	$3a+2=5b$	$\dfrac{3}{2}a+1=\dfrac{5}{2}b$	1M	for putting b on one side
	$b=\dfrac{3a+2}{5}$	$b=\dfrac{3a+2}{5}$	1A	
		---------(3)	
2.	$\dfrac{x^2y^{-3}}{(x^3y^{-1})^6}$		
	$=\dfrac{x^2y^{-3}}{x^{18}y^{-6}}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
	$=\dfrac{y^{-3+6}}{x^{18-2}}$	1M	for $c^{-p}=\dfrac{1}{c^p}$ or $\dfrac{c^p}{c^q}=c^{p-q}$
	$=\dfrac{y^3}{x^{16}}$	1A	
		---------(3)	
3.	(a) 38.2	1A	
	(b) Percentage error		
	$=\dfrac{38.26-38.2}{38.26}\times 100\%$	1M	
	$\approx 0.157\%$	1A	
		---------(3)	
4.	(a) $8m^3-4m^2n$		
	$=4m^2(2m-n)$	1A	
	(b) $8m^3-4m^2n-18mn^2+9n^3$		
	$=4m^2(2m-n)-18mn^2+9n^3$	1M	for using the result of (a)
	$=4m^2(2m-n)-9n^2(2m-n)$		
	$=(2m-n)(4m^2-9n^2)$	1M	
	$=(2m-n)(2m+3n)(2m-3n)$	1A	or equivalent
		---------(4)	

Solution	Marks	Remarks
5. (a) $5(x+2)>\frac{8x-7}{3}$		
$15(x+2)>8x-7$		
$15x+30>8x-7$		
$7x>-37$	1M	for putting x on one side
$x>-\frac{37}{7}$	1A	$x>-5\frac{2}{7}$
$6-x\ge 8$		
$x\le -2$		
Thus, the required range is $-\frac{37}{7}<x\le -2$.	1A	$-5\frac{2}{7}<x\le -2$
(b) -5 , -4 , -3 , -2	1A	
	---------(4)	
6. (a) The coordinates of A' are $(6,4)$.	1A	accept $A'(6,4)$ or $A'=(6,4)$
The coordinates of B' are $(-3,-2)$.	1A	
(b) The slope of $A'O$ $=\frac{4-0}{6-0}=\frac{2}{3}$	1M	either one
The slope of $B'O$ $=\frac{-2-0}{-3-0}=\frac{2}{3}$		accept $m_{A'O}=\frac{2}{3}$
$\because$ The slope of $A'O$ = the slope of $B'O$		
Also, O is a common point .		
$\therefore$ $A'OB'$ is a straight line .	1	f.t.
$A'O=\sqrt{(6-0)^2+(4-0)^2}=2\sqrt{13}$	1M	either one
$B'O=\sqrt{(0+3)^2+(0+2)^2}=\sqrt{13}$		
$A'B'=\sqrt{(6+3)^2+(4+2)^2}=3\sqrt{13}$		
$\because$ $A'B'=A'O+B'O$		
$\therefore$ $A'OB'$ is a straight line .	1	f.t.
	---------(4)	
7. From the given probability , we get $\frac{a}{b}=\frac{3}{5}$	1M	
$5a=3b$ ……(*)		
Also , $a-9=b-17$	1M	
$a=b-8$		
Sub. into (*) , we have $5(b-8)=3b$		
$b=20$	1A	
$a=12$	1A	
	---------(4)	

	Solution	Marks	Remarks
8.	Join AD .		
	$\angle ACD = 180° - \theta$	1A	
	$\angle ADC = 180° - \angle ABC$		
	$\angle CAD = \frac{1}{2} \times \angle COD$	1M	
	$= \frac{1}{2} \times 64°$		
	$= 32°$	1A	
	In ΔACD ,		
	$32° + 180° - \theta + 180° - \angle ABC = 180°$	1M	
	$\angle ABC = 212° - \theta$	1A	
	Join AD .		
	$\angle ACD = 180° - \theta$	1A	
	$\because$ $OC = OD$		
	$\therefore$ $\angle OCD = \frac{180° - 64°}{2}$	1M	
	$= 58°$		
	$\angle OCA = \angle OAC$		
	$= 180° - \theta - 58°$		
	$= 122° - \theta$	1A	
	$\angle AOC = 180° - 2(122° - \theta)$		
	$= 2\theta - 64°$		
	reflex $\angle AOC = 360° - (2\theta - 64°)$		
	$= 424° - 2\theta$		
	$\angle ABC = \frac{1}{2} \times$ reflex $\angle AOC$	1M	
	$= \frac{1}{2}(424° - 2\theta)$		
	$= 212° - \theta$	1A	
		---------(5)	
9.	(a) $C = as + bs^2$, where a , b are non-zero constants .	1A	
	Sub. $s = 4$, $C = 20$ and $s = 6$, $C = 36$, we have		
	$20 = 4a + 16b$	1M	either one
	$a + 4b = 5$ ……(1)		
	$36 = 6a + 36b$		
	$a + 6b = 6$ ……(2)		
	Solving (1) and (2) , we have $a = 3$ and $b = \frac{1}{2}$.	1A	for both correct
	$\therefore$ $C = 3s + \frac{1}{2}s^2$		
	(b) $3s + \frac{1}{2}s^2 = 45.5$		
	$s^2 + 6s - 91 = 0$	1M	
	$(s + 13)(s - 7) = 0$		
	$\therefore$ $s = -13$ (rejected) or $s = 7$	1A	for both correct
	Thus , the perimeter of the advertising board is 7 m .		
		---------(5)	

Solution	Marks	Remarks
10. (a) $\frac{426+20+a}{18}=25$	1M	
$a=4$	1A	
Let n be the mean age of the three new players ,		
then $\frac{25\times18-33-33+3n}{19}=24$	1M	
$n=24$	1A	
Thus , the mean age of the three new players is 24 .		
	---------(4)	
(b) As the mean age of the three new players is 24 , there are four cases :		
(1) 2 data are less than 24 and 1 datum is greater than 24 , then $m=23$;	1M	consider at least two cases
(2) 1 datum is less than 24 , 1 datum is equal to 24 and 1 datum is greater than 24 , then $m=24$;		
(3) 1 datum is less than 24 and 2 data are greater than 24 , then $m=24$;		
(4) 3 data are equal to 24 , then $m=24$.		
$\therefore$ The possible values of m are 23 and 24 .	1A	
	---------(2)	
11. (a) $\mathrm{f}(x)=(x^2-2x-3)(4x+5)+6x+k$		
$\mathrm{f}(2)=(4-4-3)(8+5)+12+k=-21$	1M	
$k=6$	1A	
	---------(2)	
(b) $\mathrm{f}(x)=0$		
$(x^2-2x-3)(4x+5)+6x+6=0$		
$(x-3)(x+1)(4x+5)+6(x+1)=0$		
$(x+1)[(x-3)(4x+5)+6]=0$	1M	
$(x+1)(4x^2-7x-9)=0$	1A	
$4x^3-3x^2-16x-9=0$ $(x+1)(4x^2-7x-9)=0$	1M+1A	
$x=-1$ or $x=\frac{7\pm\sqrt{193}}{8}$ (which are not rational numbers)	1A	
Thus , the claim is disagreed .	1A	f.t.
	---------(4)	

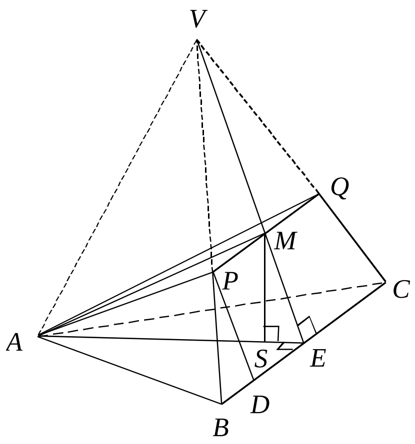
Solution	Marks	Remarks
12. (a) $\because$ $AB = BC$ (sides of a square)		property of square
$BE = CF$ (given)		
$\therefore$ $AB + BE = BC + CF$		
i.e. $AE = BF$		
And $AD = BA$ (sides of a square)		property of square
$\angle DAE = \angle ABF$ (angles of a square)		property of square
$\therefore$ $\Delta ADE \cong \Delta BAF$ (SAS)		
Marking Scheme :		
Case 1 Any correct proof with correct reasons .	2	
Case 2 Any correct proof without reasons .	1	
	---------(2)	
(b) (i) $AE = 6 + 2 = 8$		
The area of ΔADE $= \dfrac{8 \times 6}{2} = 24\ \text{cm}^2$	1A	
(ii) Construct $AN \perp DE$ such that N is the foot of the perpendicular .		
Then AN is the shortest distance from A to DE .	1M	
$DE = \sqrt{8^2 + 6^2} = 10$		
$\dfrac{10 \times AN}{2} = 24$	1M	
$AN = 4.8$	1A	
The shortest distance from A to DE is 4.8 cm .		
Therefore , there does not exist a point K lying on DE such that the distance between A and K is less than 4.8 cm .	1A	f.t.
	---------(5)	

Solution	Marks	Remarks
13. (a) $\because$ The graph of C_1 touches the positive x-axis .		
$\therefore$ $\Delta = k^2 - 144 = 0$	1M	
$k = 12$ (rejected) or $k = -12$	1A	
	---------(2)	
(b) (i) The coordinates of the point M are $(6, 0)$.		
The x-coordinate of the point R is 3 .		for both correct
Sub. into C_1 ,		
the y-coordinate of the point R is 9 .	1A	
$C_2 : y = p(x-3)^2 + 9$	1M	
sub. $(6, 0)$, we have		or sub. $(0, 0)$
$0 = 9p + 9$		
$p = -1$		
$\therefore$ $p = -1$, $q = 6$, $r = 0$	1A	for all correct
$C_2 : y = px(x-6)$	1M	accept $y = kx(x-6)$
Sub. $(3, 9)$, we have		
$9 = -9p$		
$p = -1$		
$C_2 : y = -x(x-6)$		
$y = -x^2 + 6x$		
$\therefore$ $p = -1$, $q = 6$, $r = 0$	1A	for all correct
(ii) The coordinates of the point N are $(0, 36)$.		
The area of $\Delta MNO = \dfrac{36 \times 6}{2} = 108$	1A	
Construct $RS \perp OM$ such that S is the foot of the perpendicular.		
$RS = 9$, $OS = SM = 3$		
The area of ΔMNR		
$= 108 - \dfrac{(36+9) \times 3}{2} - \dfrac{9 \times 3}{2}$	1M	
$= 27$		
$= \dfrac{1}{4} \times 108$		
$= \dfrac{1}{4} \times$ the area of ΔMNO		
Thus , the claim is agreed .	1A	f.t.
	------(6)	

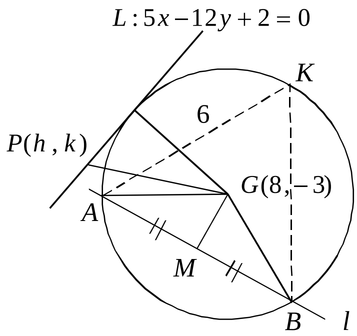
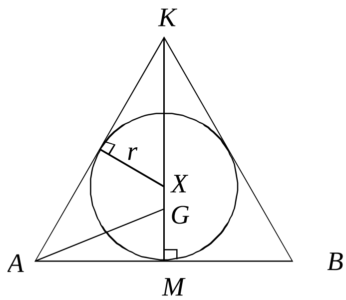
Solution	Marks	Remarks
14. (a) The curved surface area of the vessel		
$= \pi \times 10 \times \sqrt{10^2 + 24^2}$	1M	accept $\pi \times 10 \times 26$
$= 260\pi \text{ (cm}^2\text{)}$	1A	
The area of the wet curved surface of the vessel		
$= 260\pi \times (\sqrt[3]{\frac{64}{125}})^2$	1M	
$= 260\pi \times \frac{16}{25}$		
$= \frac{832}{5}\pi \text{ (cm}^2\text{)}$	1A	accept $166\frac{2}{5}\pi$ or 166.4π
The radius of water surface		
$= 10 \times \sqrt[3]{\frac{64}{125}}$	1M	either one
$= 8$		
The depth of water		
$= 24 \times \sqrt[3]{\frac{64}{125}}$		
$= \frac{96}{5}$		
The area of the wet curved surface of the vessel		
$= \pi \times 8 \times \sqrt{8^2 + (\frac{96}{5})^2}$	1M+1A	
$= \pi \times 8 \times \frac{104}{5}$		
$= \frac{832}{5}\pi \text{ (cm}^2\text{)}$	1A	
	---------(4)	
(b) Let h cm be the depth of water in the vessel .		
Then $\frac{260\pi - \frac{832}{5}\pi}{260\pi} = (\frac{24-h}{24})^2$	1M+1A	
$\frac{468}{1300} = (\frac{24-h}{24})^2$		
$\frac{9}{25} = (\frac{24-h}{24})^2$		
$\frac{24-h}{24} = \frac{3}{5}$	1A	
$h = 9.6$		
< 14.8		
Thus , the claim is disagreed .	1A	f.t.
	---------(4)	

Solution	Marks	Remarks
15. (a) The required number		
$= P_1^5 \cdot P_5^5$		accept $C_1^5 \cdot P_5^5$
$= 600$	1A	
	---------(1)	
(b) The required number		
$= P_5^5 + P_1^3 \cdot P_1^4 \cdot P_4^4$	1M	accept $P_5^5 + C_1^3 \cdot C_1^4 \cdot P_4^4$
$= 408$	1A	
The required number		
$= 600 - P_1^2 \cdot P_1^4 \cdot P_4^4$	1M	accept $600 - C_1^2 \cdot C_1^4 \cdot P_4^4$
$= 408$	1A	
	---------(2)	
16. (a) Let a and r be the first term and the common ratio of the sequence respectively .		
Then $ar = 600 \quad \cdots\cdots(1)$		
Also, $\dfrac{a}{1-r} = 3200 \quad \cdots\cdots(2)$		
$\dfrac{(1)}{(2)}$, we get $r(1-r) = \dfrac{3}{16}$	1M	
$16r^2 - 16r + 3 = 0$		
$(4r-1)(4r-3) = 0$		
$r = \dfrac{1}{4}$ or $r = \dfrac{3}{4}$		
When $r = \dfrac{1}{4}$, $a = 2400$ (rejected)		
When $r = \dfrac{3}{4}$, $a = 800$		
Thus , the first term is 800 .	1A	
	---------(2)	
(b) $800(\dfrac{3}{4})^n + 800(\dfrac{3}{4})^{2n} > 100$	1M	
$8(0.75^n)^2 + 8(0.75^n) - 1 > 0$		
$0.75^n < \dfrac{-8-\sqrt{96}}{16}$ (rejected) or $0.75^n > \dfrac{-8+\sqrt{96}}{16}$		
$\log(0.75^n) > \log\dfrac{-8+\sqrt{96}}{16}$	1M	
$n\log 0.75 > \log\dfrac{-8+\sqrt{96}}{16}$		
$n < 7.598445697$		
Since n is a positive integer .		
$\therefore$ The greatest value of n is 7 .	1A	
	---------(3)	

	Solution	Marks	Remarks
17. (a)	(i) $L_3 : x+y=30$	1A	
	(ii) $x+y\geq 5$ and $x+y\leq 30$	1A	for both correct
		---------(2)	
(b)	(i) The number of cakes allocated to Cindy from workshop B $=25-(30-x-y)$ $=x+y-5$	1A	
	The number of cakes allocated to Cindy from workshop B $=60-(20-x)-(45-y)$ $=x+y-5$	1A	
	(ii) $\begin{cases}0\leq x\leq 20\\0\leq y\leq 30\\0\leq x+y-5\leq 25\end{cases}$	1A	($y\leq 30$ can be omitted)
	i.e. $\begin{cases}0\leq x\leq 20\\0\leq y\leq 30\\5\leq x+y\leq 30\end{cases}$		
	The total transportation charge $f(x\,,\,y)=\$5[8x+4y+30-x-y+2(20-x)+(45-y)+x+y-5]$ $=\$(30x+15y+550)$	1M	
	$f(0\,,5)=625$; $f(0\,,30)=1000$; $f(20\,,10)=1300$; $f(20\,,0)=1150$; $f(5\,,0)=700$	1M	
	Thus, the least value of the total transportation charge is \$625 .	1A	
		---------(5)	

	Solution	Marks	Remarks
18. (a)	$\angle APB = 180° - 78° - 60°$ $= 42°$ In ΔAPB ,		
	$\dfrac{12}{\sin 42°} = \dfrac{PB}{\sin 60°}$	1M	
	$PB \approx 15.53105589$ ≈ 15.5 (cm)	1A --------(2)	r.t. 15.5 cm
(b) (i)			
	Let M be the mid-point of PQ . Construct $PD \perp BC$ and $ME \perp BC$ such that D and E are the feet of the perpendicular . Join AE .		
	$ME = PD$ $= PB \sin 78°$ $\approx 15.53105589 \sin 78°$ ≈ 15.19166506	1M	
	E is the mid-point of BC . $BE = 6$		
	$VE = 6 \tan 78°$ ≈ 28.22778066 $VA = VB$ $= \dfrac{6}{\cos 78°}$ ≈ 28.85840607	1M	either one
	In rt.ΔABE , $AE = \sqrt{12^2 - 6^2}$ $= \sqrt{108}$ In ΔAVE ,		
	$\cos \angle VEA \approx \dfrac{28.22778066^2 + 108 - 28.85840607^2}{2(28.22778066)(\sqrt{108})}$ $\angle VEA \approx 82.95091613°$	1M	
	Construct $MS \perp AE$ such that E is the foot of the perpendicular . Join AM .		
	$\alpha = \angle MAS$	1M	for identifying the required angle
	$MS = ME \sin \angle VEA$ $\approx 15.19166506 \sin 82.95091613°$ ≈ 15.07683711		

Solution	Marks	Remarks
$AS = AE - SE$		
$\approx \sqrt{108} - 15.19166506 \cos 82.95091613°$		
≈ 8.527989965		
In rt.ΔMAS ,		
$\tan\alpha = \dfrac{MS}{AS}$		
$\approx \dfrac{15.07683711}{8.527989965}$		
$\alpha \approx 60.50597106°$		
$\approx 60.5°$	1A	r.t. $60.5°$
(ii) $\because$ $AM \perp PQ$		
$\therefore$ AM is the line of greatest slope on the plane APQ .		
$\therefore$ α is the greatest inclination .	1M	
That is, $\alpha > \beta$.		$\beta \approx 59.3°$
Thus , the claim is disagreed .	1A	f.t.
	--------(7)	

Solution	Marks	Remarks
19. (a) $GP^2=(h-8)^2+(k+3)^2$		
$=(\frac{12k-2}{5}-8)^2+(k+3)^2$	1M	
$=\frac{144k^2-1008k+1764}{25}+k^2+6k+9$		
$=\frac{169}{25}k^2-\frac{858}{25}k+\frac{1989}{25}$	1A	
$=\frac{169}{25}(k^2-\frac{66}{33}k)+\frac{1989}{25}$		
$=\frac{169}{25}(k^2-\frac{66}{13}k+\frac{1089}{169}-\frac{1089}{169})+\frac{1989}{25}$	1M	
$=\frac{169}{25}(k-\frac{33}{13})^2+36$		
When $k=\frac{33}{13}$, the minimum value of GP^2 is 36 .	1A	
	---------(4)	
(b) (i) The equation of the circle C is $(x-8)^2+(y+3)^2=36$	1A	or $x^2+y^2-16x+6y+37=0$
(ii) Let M be the mid-point of AB . Then $GM\perp AB$.		
$GA=6$, $AM=4\sqrt{2}$		
$\sin\angle AGM=\frac{4\sqrt{2}}{6}$		
$\angle AGM=70.52877937°$		
$\angle AKB=\frac{2\times\angle AGM}{2}$		
$=\angle AGM$		
$\approx 70.52877937°$		
$\approx 70.5°$	1A	r.t. $70.5°$
or $\angle AKB\approx 180°-70.52877937°$		
$\approx 109.4712206°$		
$\approx 109°$	1A	r.t. $109°$
When ΔABK is an acute-angled isosceles triangle , KGM is a straight line and $KGM\perp AB$.		
$GM=\sqrt{6^2-(4\sqrt{2})^2}$	1M	
$=2$		
$KM=6+2=8$	1A	

Solution	Marks	Remarks
Let X be the centre and r be the radius of the inscribed circle .		
$\angle AKX = \frac{1}{2}\angle AKB$		
$\approx \frac{1}{2}\times 70.52877937°$		
$\approx 35.26438969°$		
$\because \quad KM = KX + XM = 8$		
$\therefore \quad \frac{r}{\sin 35.26438969°} + r \approx 8$	1M	
$r \approx \frac{8}{1+\frac{1}{\sin 35.26438969°}}$		
≈ 2.928203231		
≈ 2.93	1A	r.t. 2.93
The radius of the inscribed circle is 2.93 .	--------(7)	